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PROGRESS REPORT: 1980-81

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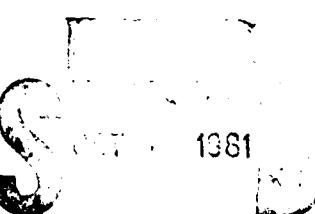
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PROGRESS REPORT AND TOPICAL SURVEY

1. Introduction

During the 1980-81 academic year part of the work at the ONR Project Center for Research on Organizational Efficiency has been focused on the application of results from game theory. Although this is only one of the several approaches to organizational design in the Center's long term program of research, the presence this year of Robert Aumann and Paul Milgrom made it an opportune time to pursue this subject. (Professor Aumann, visiting from the Hebrew University of Jerusalem, is a Senior Research Associate of the Center; and Professor Milgrom, visiting from Northwestern University, is a Research Associate.) In addition, Kenneth Arrow and Robert Wilson of the Center's permanent staff have current research based on the use of game-theoretic models.

For this survey we have selected the topic of dynamic games with incomplete information. Our aim is to describe the main lines of investigation developed during the year and relate them to the general problem of organizational design. Included as well are connections with the wider literature on game theory, economics, and organizational efficiency.

The theme of this year's research is multi-person decisions affected by subtle considerations of timing and information. Interactive decisions among the members of a team or an organization often have, of course, important intertemporal aspects; and at the heart of organizational activity is the acquisition and dissemination of information. Much of the analysis of organizational design treats the decision process as a cooperative endeavor motivated by common interests and goals. Nevertheless, fundamentally the feasibility of cooperative behavior depends critically upon the provision of

incentives to meld conflicting interests into shared concerns. The study of noncooperative or competitive multi-person decision problems is the foundation for constructive results about the design of organizations.

Our research program, therefore, has two main components. One is basic research on the theory, modeling, and analysis of dynamic games with incomplete information dispersed among the participants. Here the central issue is the role of timing, information, and communication. The second component is the study of incentives and other inducements to coordination of actions and the attainment of organizational efficiency. Most of the substantive accomplishments this year are on the subject of dynamic games; in addition, we shall take the opportunity to survey more widely the recent results by others working on this topic.

DYNAMIC GAMES WITH INCOMPLETE INFORMATION

Dynamic games with incomplete information in which the precise details of timing, information, and observation are modeled explicitly are known technically as extensive games. Our work has included both the study of general extensive games, and a special class called repeated games that model decision processes that continually recur. These two topics are described separately below. Additionally, a third section describes our recent work on a related problem: namely, under what circumstances can one ensure the existence of optimal strategies that do not require randomization? From a practical viewpoint nonrandomized strategies are an important desideratum of a viable and practically useful theory of multi-person decisions.

1. Extensive Games

The organizing principle of game-theoretic models of multi-person decision processes is the concept of an equilibrium, as formulated by Nash. For the players' strategies to constitute an equilibrium requires that each player's strategy is optimal (for that player) given the others'; that is, each player's strategy is a best response to the others' strategies. Recent research, initiated by Selten, has indicated, however, that this criterion is insufficiently stringent to capture fully the effects of timing and information in dynamic games. The following example devised by Selten depicts the difficulty in its simplest form (see Figure 1).

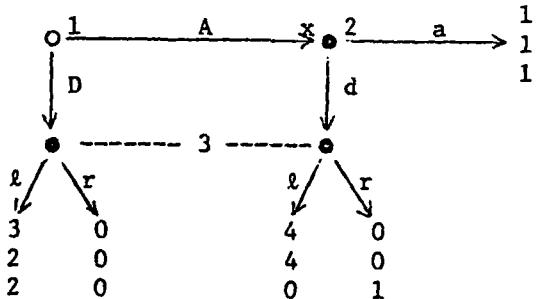


Figure 1*

* In these depictions of extensive games, nodes $x \in X$ are indicated by circles, open circles in the case of initial nodes. Terminal nodes are indicated by the vector payoffs that go with them, with player 1's payoff given first, 2's second, and so on. Precedence is depicted by solid arrows, with the arrow pointing towards the successor. Actions are labelled on the arrows. Information sets that are not singletons are indicated by dashed lines joining the nodes that make up the information set. Numbers at nodes or in information sets indicate the player whose turn it is to choose an action. When necessary, initial assessments ρ are depicted by probabilities in braces at the initial nodes.

One of the Nash equilibria in this game has players 1, 2, and 3 all moving right, but there is another in which players 1 and 3 reverse their choices. This second equilibrium is open to the criticism that 2 would, if given the opportunity by 1, prefer to reverse his choice too (since he anticipates 3's move to the left, which is optimal for 3 when he is expecting 1 to move down). The feature that sustains this "imperfect" equilibrium is the fact that any strategy is optimal for 2 given his anticipation that 1 and 3's strategies will not present him with the opportunity to move.

Selten has proposed a criterion for perfect equilibrium that is stronger than Nash's criterion. Selten's criterion requires not only that each player's strategy is a best response to the others' strategies, but also that it is robust. Robustness requires that a player's strategy remains optimal even if the other players' strategies are perturbed in a specific way so that every move has positive probability of being taken. Selten's criterion is marvelously effective in eliminating extraneous equilibria, as in the example above. It is, however, quite difficult to apply since the determination of the required perturbations is not amenable to easy computations.

Seeking to improve Selten's construction, Kreps and Wilson, in a Technical Report of the Center, have devised a weaker form of Selten's criterion that brings with it some valuable insights into the role of beliefs and expectations in interactive decisions. Kreps and Wilson impose the criterion of sequential rationality and call an equilibrium satisfying this criterion a sequential equilibrium.

Simply stated, sequential rationality requires that every decision by a player must be part of an optimal strategy for the remainder of the game. (Kreps and Wilson show that for "almost all" finite games, namely generically in the payoffs, the perfect and sequential equilibria are the same.) The thrust of

this criterion is that at each juncture the decision maker's strategy must be an optimal solution to the dynamic programming problem reflecting the continuation of the game, with respect to some probability assessment over the feasible histories of the course of play so far. Their paper includes a treatment of the Bayesian consistency conditions required of the probability assessments, an extensive review of the connection to Selten's criterion, and a study of the generic properties of sequential equilibria. We expect the paper to have a major impact on the study of extensive games.

Here we concentrate mainly on the ramifications for the study of organizational design. We interpret an extensive game as representing schematically a sequence of decisions made by the members of an organization, admitting the possibility that there may be some conflict among their interests. An equilibrium, then, is a plan about how they will behave in various contingencies as time evolves — with the property that no member has an incentive to subvert the plan by not following his prescribed course of action. The plan, in other words, is incentive compatible. The criticism raised by Selten is that, in a Nash equilibrium, one member's plan is incentive compatible only if the other members adhere to their parts of the overall plan. In contrast, Selten imposes the stronger criterion of robustness, admitting small probabilities that other members may tremble, falter, or otherwise deviate from the plan. Kreps and Wilson weaken Selten's criterion (slightly, as it turns out) to say only that in the unexpected event that other members deviate, a member's plan of action from that point forward should be optimal if other members subsequently adhere to their original plans contingent on that deviation.

We see the viewpoint espoused by Selten, and by Kreps and Wilson, as an important step in bringing greater realism to the predictions of game theory, by excluding equilibria based on anticipated behavior in unexpected circumstances

that does not meet the criterion of rationality should those circumstances somehow arise. In the game depicted in Figure 2, for example, the only optimal move for player 2 (if 1 gives him the opportunity) is left, so 1 will indeed want to move left too, and this equilibrium is perfect and sequential. (The excluded equilibrium has 1 moving across and 2 moving right.)

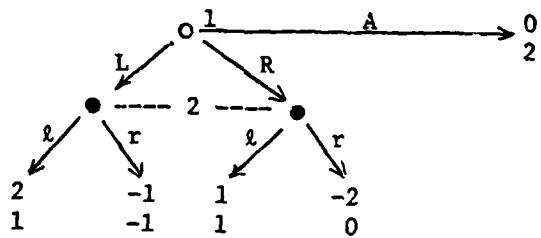


Figure 2

The important consequence of these developments is only in part the enforcement of optimal behavior in unexpected events. Underlying this criterion is the need to specify the players' beliefs should the unexpected

occur. In Figure 3, for example, player 2's probability assessment as to whether 1 has chosen left or right is critical to 1's choice of whether to give 2 the opportunity to move. In general (as elaborated in Kreps and

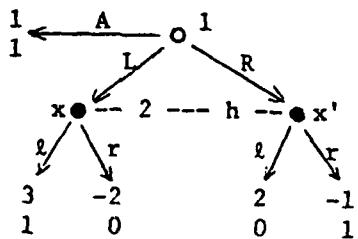


Figure 3

Wilson's paper) a plan of joint actions that is to be sequentially rational must be supported by a pattern of beliefs among the members specifying in each unforeseen contingency the relative probabilities about how that contingency might have occurred (e.g., in Figure 3, how 2's information set might have been reached).

The second development of significance is the discovery that restricting the Nash equilibria to those that are sequentially rational is the requisite constraint enabling each member to solve his own personal decision problem by the ordinary methods of backward recursion (dynamic programming) through his personal decision tree induced by the game tree. That is, in principle the familiar methods of decision analysis are applicable to the optimization of each member's strategy.

We see this work as providing the foundation for the constructive application of the methods of game theory to multi-person decision problems — since sequential equilibria can be factored into individually optimal strategies. Further, this development is the missing link in the analysis of the possibilities of decentralizing organizational decision processes affected by timing and information — since it allows the general plan to be examined as the composition of plans that are optimal for subunits, and in turn, for individuals. Decomposition in time is also possible — since at any juncture in midgame the subsequent plan remains optimal for the remainder of the game.

In ongoing work we intend to develop further the applications of sequential equilibrium to problems of organizational design. Besides the topics on decentralization afforded by this new perspective, we are particularly interested in elaborating how to specify beliefs so as to meet basic tests of consistency and plausibility (Kreps and Wilson, Section 8).

2. Finitely Repeated Games

In other work* by Kreps and Wilson they have applied the theory of sequential equilibria to a class of two-player finitely-repeated games with incomplete information. That is, the same stage game is repeated many times, but which among several possible stage games is being repeated is uncertain to one or both players. In the class studied, a player's uncertainty is only about what are the other player's preferences.

A central issue in such games is whether it is advantageous for a player to dissimulate, namely, to behave "as though" his preferences are other than they actually are, or indeed even to behave "irrationally".

The standard example that motivates this issue is the repeated version of the stage game depicted in Figure 4.

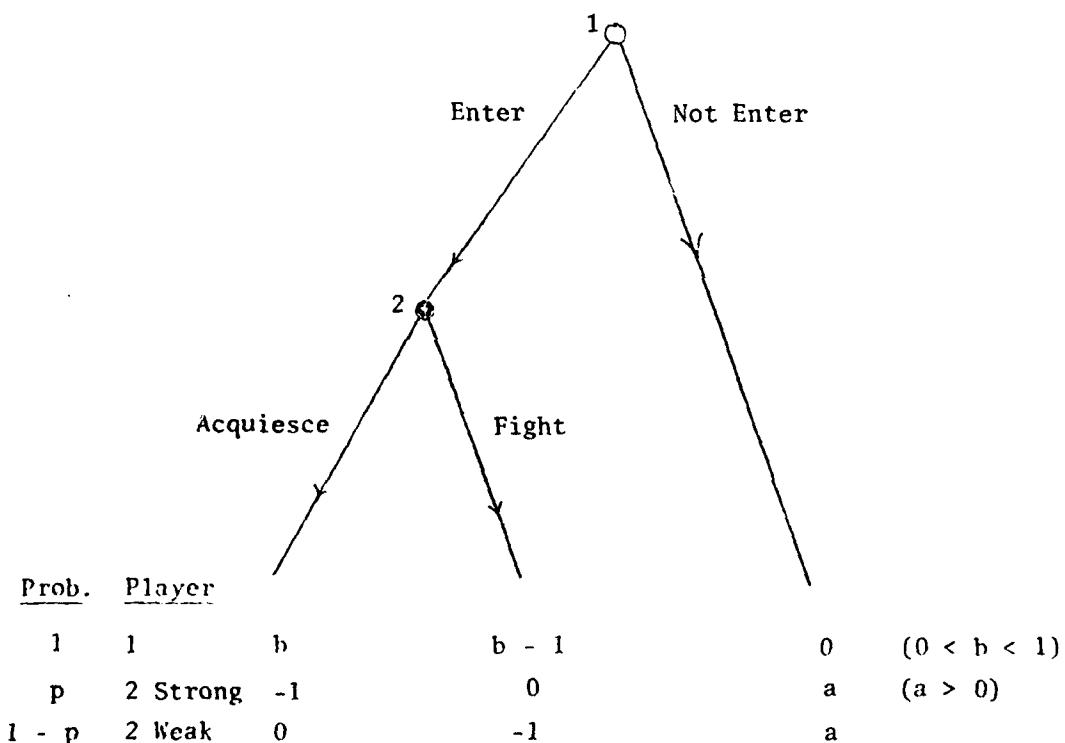


Figure 4

* Supported through August 1980 by the National Science Foundation.

Player 1 is uncertain about 2's preferences: he might be Strong with probability p , or Weak. They are to play the game n times (with 2's type the same each time) and 1 gets to observe 2's choices but not 2's payoffs; each player's payoff is the sum of the n stage-game payoffs. How would a Weak type of player 2 behave? If $p = 0$ then of course 1 always enters and 2 always acquiesces. But if p is positive and n is sufficiently large, it turns out that matters are much different! Kreps and Wilson construct a sequential equilibrium in which, if the number of stages remaining exceeds $1 + \log(b/p)$, then (a) either type of player 2 is prepared to fight, and consequently (b) player 1 never enters. Thus, if the number n of stages is large, the Weak type of player 2 succeeds nearly as well as the Strong in deterring 1's entry.

The ramifications of this result are quite startling. In recurring situations with many repetitions, a player with privately known preferences or technology (Weak) can successfully imitate the behavior of one with a more advantageous technology (Strong). In Kreps and Wilson, and a related paper by Milgrom and Roberts, this feature is amplified to a more general theory of a player's "reputation" as a major explanatory hypothesis for strategic behavior. The idea, in brief, is that a player's reputation is summarized by other players' probability assessments about what his preferences (costs, technology, opportunities, etc.) might be, which are known privately by the player. The import of the analysis is to demonstrate that maintenance or enhancement of his reputation, in the sense that his behavior sustains or increases the probability that he is in a favorable position, may be sufficient incentive for him to adopt behavior that would otherwise appear non-optimal. That such reputation-enhancing strategies constitute a perfect or sequential equilibrium satisfying sequential rationality is the insightful and innovative contribution of this work.

One of the major applications of this view is presented in the second part of the Kreps-Wilson paper. It presents an explanatory theory for episodes of intense competition, such as wars of attrition, price wars, battles for market share. The basic model is the same as Figure 4 except also 2 is uncertain about 1's payoffs, namely whether b is positive (Strong) or negative (Weak). The equilibrium play between the two Weak players is shown to take the following form. Both players act strong (enter and fight, respectively) for a duration (privately known to each, and possibly randomly determined) after which each is prepared to accede to the other (not enter, or acquiesce) if the other has not previously acceded. Thus, the end result is that a battle ensues for a period, equal to the minimum of the two players' maximum durations of aggressive behavior, after which one or the other capitulates, still not knowing whether his opponent is truly Strong or Weak. By formulating this game in continuous time, moreover, Kreps and Wilson obtain an elegant closed-form solution.

Subsequent work on this topic has valuable potential to elucidate competitive episodes that heretofore have had few satisfactory explanations. The key element here, as in much of the other recent work, is the central role of the complex dynamic processes engendered by the features of timing and information.

3. Infinitely Repeated Games

Infinitely repeated games with incomplete information are being studied because they provide well-specified models in which communication among the members of an organization can be subjected to precise analysis. The general phenomena of communication are not easily captured in tractable models due to the difficulties of capturing the salient aspects of syntax and semantics. In repeated games, however, the moves of the players have natural interpretations as signals, and the language of communication is entirely summarized in the histories of moves. The focus on infinitely-repeated games with an average-payoff criterion, moreover, ensures that any finite sequence of moves intended for communication purposes has no effect on the long-run average payoffs attained by the players. Thus, the single model encapsulates a language for communication as well as the decision problem that is the subject of the communication.

This line of work is being pursued by Robert Aumann, with contributions by Sergiu Hart and Sylvain Sorin. Since the general theoretical development has not yet been expositioned in a Technical Report of the ONR Project Center, here we set forth a simple version that conveys the main elements. Also, we use an example to illustrate the key ideas.

We consider a typical problem of coordination between two members of a team with somewhat differing interests. The significant feature is that one player (Column) is uncertain about the other's (Row's) preferences. In particular, the game to be played is one of the two bimatrix games depicted below:

Payoffs to Row, Column					
Game 1			Game 2		
Row/Col.	Left	Right	Left	Right	
Up	2,1	0,0	2,1	2,0	
Down	0,0	1,2	0,0	1,2	

Row knows which game is played whereas Column initially assigns probability p to Game 2. At first glance one observes that Row could simply "assert" Game 2 to be the one in either event, and then play Up forever (his dominant strategy in Game 2), forcing Column to play Left, yielding payoffs of (2,1) to Row and Column. We are interested, however, in knowing all the equilibrium payoffs. For instance, if Column knew Game 1 to prevail (*i.e.*, $p = 0$) then any payoff pair that is a convex combination of (2,1) and (1,2) is attainable by equilibrium play. We shall see, however, that Column's uncertainty severely restricts the attainable set of equilibrium payoffs. Indeed, for p positive but approaching zero the equilibrium payoffs are the set of convex combinations of (2,1) and (4/3, 5/3). To show this, we study the equilibrium that yields expected payoffs (4/3, 5/3) in Game 1 and (2, 2/3) in Game 2.

To simplify matters we merely assume that the players have a language of symbols S by which Row can communicate to Column; actually, however, S arises as the set of finite histories of moves by Row augmented by a dictionary that describes how any history is to be interpreted. The analysis of this language has been developed by Hart who shows the exact mechanism by which Column uses observed histories to calculate posterior probabilities for the two games using Bayes' rule. Presently it suffices that $S = \{1,2\}$, so that Row merely signals that the game is either Game 1 or Game 2.

Moreover, it suffices to describe equilibrium play summarily as an enforceable joint plan. Such a plan is a pair (q, f) consisting of a signalling

strategy q for Row, and a contract f . The signalling strategy for Row specifies the probabilities q_{ij} that Row will signal j when the game is i . And the contract f specifies the frequencies f_{kl}^j that they will play the strategy pair (k,l) when the signal is j . The proposed equilibrium is described below in these terms:

	<u>q</u>		<u>(f^1, f^2)</u>	
<u>i/j</u> :	<u>1</u>	<u>2</u>	<u>Left</u>	<u>Right</u>
1 :	1	0	Up: $1/3, 2/3$	$0, 1/3$
2 :	0	1	Down: $0, 0$	$2/3, 0$

This plan achieves the expected payoffs mentioned earlier.

To see that this is an equilibrium, we first note that Row's signalling strategy is to signal truthfully. Accepting this, therefore, Column can do no better in Game 1 than to coordinate with Row — every third time is Up, Left; the others, Down, Right — since these are both equilibria of the stage game. In Game 2, however, Row always plays Up and Column is supposed to alternate between coordination (Left, two-thirds) and not (Right, one-third). The latter is not optimal in the short term, but it can be enforced by Row using a maxmin strategy (Up, two-thirds; Down, one-third), should Column defect, that keeps Column's payoff at $2/3$, which is what Column gets by adhering to the contract. For exactly analogous reasons, Row's adherence to the contract is enforceable by Column. Given the enforceability of the contract, Row's signalling strategy is optimal: in Game 1 adherence yields payoffs 2 and 1 with frequencies $1/3$ and $2/3$ for an average payoff of $4/3$, which is what he would get by falsely signalling Game 2; and in Game 2 he gets his best payoff of 2. Thus, we see that this plan is enforceable as an equilibrium in the infinitely repeated game. The assertion that no other equilibrium yields more to Column if Game 2 has positive probability follows from the observation that in Game 1

Row is just indifferent whether to signal truthfully; any greater payoff for Column would entice Row to deviate from the truthful strategy of communication about which game obtains.

In the general case an enforceable joint plan is described by a signalling strategy for each player and a subsequent contract. Given the contract, the signalling strategies must be in equilibrium; that is, be self-enforcing. The contract must satisfy individual rationality for each player; that is, the contract must give an average payoff not less than what the player could enforce by using a maxmin or Blackwell approachability strategy in the infinitely repeated game.

In work to date Aumann has been developing the theory of enforceable joint plans in general, infinitely-repeated games with incomplete information. It has been known for some time, that the payoffs to each enforceable joint plan are obtainable as payoffs to an equilibrium in the infinitely repeated game. The converse, however, he has shown to be false. Sorin has recently shown that the set of enforceable joint plans is never empty if only one player has incomplete information. The connection between the signalling language S and the set of finite histories of moves has been developed in companion work by Hart. At present none of these contributions has been prepared for publication.

The ramifications of this research for problems in the theory of organizational design are quite important. Besides providing a tractable model that allows precise analysis of the rudiments of communication, the work highlights the dramatic effects of even slight imperfections of information. In the example above, for instance, any positive probability for Game 2, however small, excludes the possibility that Row can get less than $4/3$ as an equilibrium

payoff in Game 1; moreover, in Game 2 the inefficient outcome (Up, Right) must occur with positive frequency. One insight afforded by these results is that there are large gains obtainable from using incentives and information systems in organizations to ensure an identity of interests and/or information among the members of a team. Another is the possibility of self-serving dissimulation as a key factor constraining the attainable set of equilibrium outcomes.

4. Nonrandomized Strategies

To the layman interested in multiperson decision problems, the baffling aspect of the game-theoretic studies is the recourse to randomized strategies. That is, equilibria of games are often constructed on the supposition that a player may "flip a coin" to decide between two actions between which he is otherwise indifferent. In the theoretical studies, this approach is often taken in order to ensure easily that the desired equilibria will exist.

In fact, however, most well-formulated models of strategic interactions do not require randomization to ensure the existence of equilibria. This fact has been recognized informally for some years, dating back at least to Bellman and Blackwell in 1949.

This proposition is, moreover, of enormous practical importance for the application and uses of game theory. In practical circumstances, in deciding matters of great importance, decision makers are loathe to act on a recommendation to randomize. Moreover, randomization runs counter to most of the received tradition of decision theory and behavioral psychology.

To be fair, one should realize that the game-theoretic device of randomization is merely an abstract version of the notion that the choice is to be based on further, unspecified information. It is only for analytical convenience that this information is construed as the private observation of an independent experiment — the flip of a coin, roll of a die, spin of a roulette wheel. In fact, nearly any source of finer information will do.

Recently, however, Professors Aumann and Milgrom, and their coworkers, following initial work by Radner and Rosenthal, have provided incisive characterizations of conditions sufficient for the existence of equilibria in pure (*i.e.*, nonrandomized) strategies.* Here we summarize briefly their main results.

* Part of this work has been supported by the Hebrew University and by the National Science Foundation.

These are of two kinds. One is an exact purification theorem, and the other is approximate, in the sense of an ϵ equilibrium (for every $\epsilon > 0$ there exists an equilibrium that gives each player within ϵ of the payoff obtainable with a best response).

The exact form was established by Milgrom and Weber and its refinements have been developed by Milgrom at the Center. Two equilibria are said to correspond if in both each player faces the same probability distribution of his environment (and thus the same decision problem). The theorem establishes sufficient conditions for every equilibrium in randomized strategies to correspond to some equilibrium in pure strategies. Thus, no player could discern, nor would he care, whether his opponents were playing a randomized or pure equilibrium. The sufficient conditions are that there is some unobserved, finite-valued state variable such that each player's information is conditionally independent, with an atomless distribution, given the state; and each player's payoff depends only on the state, his information, and the players' actions. These sufficient conditions are valuable because they encompass a wide class of the game models used to describe strategic interactions.

Their corresponding "approximate" purification theorem asserts that if the distribution of a player's information is atomless then his pure strategies are dense (using the topology of weak convergence) in his set of randomized strategies. Thus, an observer of the game would be unable to distinguish whether or not the player was randomizing if he could only observe the player's information and actions subject to some continuously distributed error. A corollary, moreover, asserts the existence of an ϵ equilibrium in pure strategies, using some additional weak hypotheses.

The "approximate" theorems of Aumann et al. are slightly different. Their first result requires that the players' action spaces are finite, and that each player's information conditional on any other player's information is atomless. It is then shown that every equilibrium has an epsilon purification — i.e., an equilibrium that is "close" (in the weak topology) and leaves every player's decision problem "almost" unchanged, so that in particular each player's corresponding pure strategy is within epsilon an optimal response to the others' strategies. The second result provides such a result for strategies, rather than equilibria, if a player's information conditional on all other players' information is atomless. In this case, every randomized strategy has a corresponding epsilon-optimal pure strategy against any strategies of the other players, and that leaves the other players' decision problems "almost" unchanged.

The cumulative implication of these results is that in a very wide class of realistic models of competitive situations, in which each player possesses some continuously distributed private information, an equilibrium (or epsilon equilibrium) can be attained. Thus, private information, sufficiently fine, is the requisite ingredient that obviates the need for randomization! It shows that sufficient care to incorporate private information into models of strategic interaction will eliminate the practical difficulties associated with interpreting and applying randomization.

5. Summary

→ The work at the Center on game-theoretic models of multiperson decision problems has strengthened the analytical tools available to study the phenomena of organizations. There are bright prospects that several of the more intractable topics can be addressed anew with better chances of clear-cut resolution. To take a specific example, we are particularly interested in developing the theory of bargaining modeled as a dynamic game with incomplete information. That is, bargaining or negotiations are typically affected by incomplete information about each other's preferences, resources, and opportunities. At the same time, bargaining is inherently a dynamic process of offers and counteroffers, perhaps interrupted by combative tactics (strikes, lockout, "war"). It seems clear that the analysis of such a model can proceed smoothly along the lines developed in work to date.

←

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OTHER RESEARCH ACTIVITIES

Kenneth J. Arrow is working on an extensive survey of the role of information in the market. There is a wide and disparate literature on this area, and the survey will attempt to bring some order into the field. Information is here interpreted as the existence and possible acquisition and transmission of signals, random variables which are correlated with unknown states of nature relevant to the payoffs to alternative courses of action by individuals. (1) One area, more studied in application than in general theory, is the acquisition of new information (research and development), and the possibilities of its being retained by its acquirer. If the information cannot be retained, there will be little incentive to acquire it; if it can, it leads to a situation of differential information. (2) There are a variety of situations in which individuals have differing information (have observed different signals). In a rational world, each knows that others have different information. In the simplest models, those of competitive bidding, each individual will realize that if he or she were to win the bid, it would be because of more favorable information than others have received; hence, the total information would be less favorable than that received, and the bid should be modified accordingly. More generally, the presence of differential information implies that any individual can infer from the behavior of others what information they have. The "signalling" literature started by Michael Spence exhibits another form of behavioral response to differential information. In this case, an individual may seek to transmit a signal favorable to him or her but has to make it credible.

It is the aim of this survey to bring the different problems in this area into some kind of order and exhibit their cross-relations.

Arrow is also completing a paper on the interactions of information and economies of scale. A firm using information has an inevitable form of economies of scale, since information (for example, about the cost of production) is needed only once regardless of the scale of the enterprise. One can postulate a whole range of possible signals with varying costs and with various degrees of precision in specifying the true optimal method of production. Presumably, high costs are needed for greater accuracy. Any given signal will be justified by a sufficiently large scale, if the firm is risk-neutral. Risk aversion might, however, be thought to set a limit to this process, since any residual uncertainty is magnified by the scale of the operations. However, a sufficient condition has been developed on the nature of the utility function and the distribution which implies that by taking a large enough scale and acquiring enough information to reduce the uncertainty correspondingly, any level of expected utility can be achieved.

Robert J. Aumann is following a recent paper of O'Neill [1] which

Robert J. Aumann is following a recent paper of O'Neill [1] which discusses problems of conflicting claims from the standpoint of rights arbitration. Specifically, suppose that each of n protagonists presents an instrument (e.g., a deed of gift) in court specifying how much of an object (e.g., an estate) he is to receive. If these instruments are inconsistent (or not necessarily consistent), what division of the object should the court make in view of all these instruments?

O'Neill suggests the following inductive procedure. When $n = 1$, the single protagonist gets as much of the object to be divided as there is. Suppose the problem has been solved up to, but not including, n protagonists. If we accept the i 'th protagonist's instrument, it is still not clear how the amount remaining of the object after he has received his claim is to be divided. For this, O'Neill applies the $(n - 1)$ -person procedure, which by the inductive procedure has already been constructed. Thus, the n instruments can be construed as providing n different prescriptions for dividing the object among the protagonists (other than just specifying how much one of the protagonists is to receive). O'Neill then gives each protagonist his expectation, using a probability of $1/n$ for each instrument. This is based on a sort of "principle of insufficient reason;" to quote O'Neill, "No will has any more validity than any other, so it should be assumed that each is equally likely to be right and has equal reason to be adopted."

This reasoning was challenged by a Talmudical scholar named Diskin living in Jerusalem more than 100 years ago. Diskin considers two cases. In the first case, two people claim a garment, one saying it is all his, and the other saying it is half his. In this case, O'Neill's method leads to a division of $3/4 - 1/4$, which coincides with the classical division prescribed in the Mishna (the basis of the Talmud) about 2,000 years ago. Suppose, now, that another claimant enters; O'Neill's method then leads to a division of $1/2 - 1/4 - 1/4$. Diskin points out that this violates the basic desideratum of strict monotonicity, since the entry of the additional claimant affects only the "larger" of the two original claimants; the "smaller" one's payoff remains intact, and the entire payoff of the new claimant comes from the larger claimant.

Diskin, therefore, rejects O'Neill's solution and proceeds as follows (in the three-person case described above): like O'Neill, he starts by "completing" the claims; thus each protagonist who claims half the garment assigns a quarter to each of the other two. One can thus describe the situation by the matrix

$$A = \begin{pmatrix} 1 & 1/4 & 1/4 \\ 0 & 1/2 & 1/4 \\ 0 & 1/4 & 1/2 \end{pmatrix}$$

whose ij 'th entry represents the amount assigned by the j 'th protagonist to the i 'th protagonist.

Diskin then reasons as follows; All agree that 1 should get $1/4$; therefore, he should first be given $1/4$, leaving us with the matrix

$$\begin{pmatrix} 3/4 & 0 & 0 \\ 0 & 1/2 & 1/4 \\ 0 & 1/4 & 1/2 \end{pmatrix}$$

On an additional $1/4$ there is no argument between 2 and 3; 3 agrees that 2 should get it. However, 1 does not agree; since 1 and 2 disagree about this $1/4$, they should split it. Similarly, another $1/4$ is split between 1 and 3. About the last $1/4$ all three disagree, and therefore it is split three ways.

Mathematically, this can be said as follows: One can decompose A into a non-negative combination of "basic" matrices, where each basic matrix is a matrix of 0's and 1's representing a situation in which some of the protagonists claim the entire garment, and the other protagonists make no claims for themselves, but agree with one or another of the claimants. In our case

$$A = 1/4 \begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + 1/4 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix} + 1/4 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 1 \end{pmatrix} + 1/4 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} .$$

A conflict represented by a basic matrix is settled by equal division among the claimants, and the original problem is solved by adding up the settlements from the "basic" components.

Unfortunately, this procedure does not lead to a unique solution. The above decomposition leads to a payoff of $(7/12, 5/24, 5/24)$. But, we can also decompose A into basic matrices as follows:

$$A = 1/4 \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} + 1/4 \begin{pmatrix} 1 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} + 1/4 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix} + 1/4 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 1 \end{pmatrix} .$$

This leads to the payoff $(1/2, 1/4, 1/4)$, which Diskin specifically rejected.

A careful reading of Diskin reveals that he was referring not just to any decomposition, but to a decomposition obtained in a fairly specific way. To describe it, let us call a basic matrix a "type j" matrix if there are j claimants, i.e. if precisely $n-j$ rows consist of zeros exclusively. Diskin's decomposition is constructed by finding a decomposition with the following properties: The type 1 basic matrices have a maximal coefficient sum (among all possible decompositions); the type 2 matrices have a maximal coefficient sum (among all possible decompositions with maximal coefficient sum for type 1 basic matrices); and so on. That is, one tries to express the given matrix as a sum of basic matrices in a way that "minimizes" conflict.

While it is still not entirely clear that this procedure does yield a unique solution, this can be shown to be the case.

It would be interesting to see whether the Diskin method can be described axiomatically.

Other ongoing work in this area involves a characterization of the O'Neill solution in terms of the Shapley value. Other solutions have also been proposed for division problems, which can also be characterized as Shapley values of appropriate characteristic function games.

Still other ongoing work involves the legal underpinning of various division schemes, and how various legal approaches and principles can be translated into mathematical terms, yielding specific division schemes.

REFERENCES

- [1] B. O'Neill, "A Problem of Rights Arbitration from the Talmud," Discussion Paper No. 445, The Center for Mathematical Studies in Economics and Management Science, Northwestern University, November 1980.

PROFESSIONAL ACTIVITIES

A major role of the Center is to promote and sustain a wide range of professional activities that contribute to research on organizational efficiency. The purpose of these activities is to stimulate scholars and practitioners from many fields of endeavor to appreciate the progress and challenges of this research area. The complex interdisciplinary character of the subject requires that criticism and contributions be obtained from several sources.

The chief modes of professional interchange are described briefly below:

Research Associates: The Senior Research Associates and Research Associates on the Center's staff are selected on the basis of outstanding contributions to research on topics connected with organizational efficiency. Most of these Associates reside at the Center for nine or twelve months. We are most fortunate to have assembled a distinguished group for the year 1980-81. Each depends upon the Center for a substantial part of his research support.

Visitors and the Lecture Series: In the course of each year several prominent scholars are invited for short periods to participate in the Center's research programs. In each case a main event is the presentation of a public lecture on a topic in the visitor's field of study. The first visitor and lecturer was William Gorman of Oxford University, for three weeks in May-June 1980.

Interdisciplinary Seminar in Decision Analysis: This Seminar draws upon the diverse talents available at Stanford University both within and without the Center. The steering committee consists of Professors Arrow and Wilson and Professor Amos Tversky of the Department of Psychology. The topics concentrate on three areas: (1) empirical and theoretical studies of hypotheses about behavior under uncertainty alternative to the expected-utility hypothesis; (2) decision behavior in organizations; and (3) studies of specific issues in actual decision-making, particularly those involving public policy.

The proceedings of these seminars, including both papers or notes of the speakers and a report on the floor discussion, are distributed annually. The speakers and topics this year have been the following:

<u>DATE</u>	<u>SPEAKER</u>	<u>TITLE</u>
October 23	Ronald Howard	The Value of Life
November 13	David Freedman	Econometric Models Don't Work
December 4	Robyn Dawes	Intuitive vs. Linear Prediction
January 15	George Dantzig	Complexity of Models and the Real World
February 12	Edward Shortliffe	Capturing and Utilizing the Judgmental Knowledge of Experts in a Medical Decision-Making System

March 12	Peter Hammond	Consistent Utilitarian Objectives Under Uncertainty and With Incomplete Information
*April 30	Thomas Cover	Decision Making With Finite Memory
*May 21	Herbert Abrams	Evaluation of Angioplastic Procedures

Conferences: It is intended that the Center occasionally sponsor a major conference on a topic central to its research programs. Presently plans are being developed for a conference, probably in 1982, on the general subject of the Economics of Information. It is hoped that such conferences will succeed in convening the major contributors to the subject for a thorough assessment of the state of the field and its significance for practical affairs.

Publications: Part of the function of the Center's staff is preparation of expository reports in the areas of the Center's work. In addition to technical reports of research accomplished by the staff, it is intended that the Center will publish reports on the field in general, designed to be useful to those involved in practical problems of evaluating and redesigning existing organizational arrangements. In addition, rapporteur's reports are prepared for the Center's various modes of professional activities. The three main report series are:

Technical Reports and Working Papers: These two series are intended for publication of completed research results, and preliminary results, respectively. Published articles are distributed as part of the Reprint Series. The Appendix lists those prepared during the current year.

Survey Papers: At infrequent intervals the Center will publish major surveys of important topics of practical interest. These are intended to cover broad fields of theory and practice, and to appeal to a wide audience. Kenneth Arrow is currently preparing surveys on two topics: "Information and the Market," and "The Theory of Social Choice."

Seminar and Lecture Reports: The Seminar reports prepared by the Center's staff include presently the papers presented and the discussions at the Seminar on Decision Analysis. Also to be included are the Lecture Series and the proceedings of conferences sponsored by the Center. The first speaker in the Lecture Series was William Gorman of Oxford University in May and June 1980. As mentioned, plans are in a formative stage for a first conference on the Economics of Information; it is hoped that the Proceedings can be prepared in book form.

The Center will in the course of its normal procedures prepare periodic Progress Reports for the Office of Naval Research describing the Center's research accomplishments and summarizing its various activities. Copies of all publications will be transmitted to the Office of Naval Research.

Advice and Assistance: As part of its professional responsibility the

*Forthcoming

Center stands ready to supply advice to the Office of Naval Research, and through it to other organizations, on the principles of evaluation and design of organizations and of information systems.

The research programs of the Center are intended to contribute broadly to studies of organizational efficiency. Among the topics to be investigated several are understood to be directly relevant to major areas of responsibility of the Office of Naval Research. These include (1) bidding, nonlinear pricing, and contracting; (2) design of efficient incentives; (3) economic analysis of information systems; (4) game-theoretic analysis of strategic behavior.

Publications of the Center will be forwarded to the Office of Naval Research, and at its discretion, made available for wider dissemination. In particular, the critical surveys of major topics provide comprehensive reviews of the state of knowledge in several areas of general importance to the research programs of the Office of Naval Research.

The several modes of professional interchanges that take place--including the Interdisciplinary Seminars, the Seminar and Lecture Series, and a major conference on the Economics of Information--will provide a valuable resource for those representatives of the ONR who choose to attend.

The Center welcomes the transmittal of comments about the choice and design of its research program, and of information about topics connected with the subject of organizational efficiency that may deserve intensive study.

APPENDIX

TECHNICAL REPORTS

T. R. 327. TWO PAPERS ON MAJORITY RULE:

1. Peter Coughlin and K. Lin, "Continuity Properties of Majority Rule with Intermediate Preferences," February 1981.

2. Peter Coughlin and S. Nitzan, "Electoral Outcomes with Probabilistic Voting and Nash Social Welfare Optima," February 1981.

ABSTRACT

This technical report contains two papers on aspects of majority rule.

The first (with Kuan-Pin Lin) studies the continuity properties of majority rule. Specifically, it shows that certain conditions which have previously been shown (by Grandmont) to be sufficient for a society's majority rule relation to be transitive or acyclic are also sufficient for the map from distributions of voter preferences to indices identified with their majority rule relations to be continuous. Applications of this result to societies with certain classical assumptions on preferences reveal that, in such societies, the map from distributions of voter preferences to their majority rule equilibria is also continuous.

The second (with Shmuel Nitzan) analyzes outcomes from electoral competitions with a Luce model of probabilistic voting. These outcomes are shown to be precisely the social alternatives that maximize a Nash-type social welfare function. These outcomes are also shown to be unanimity likelihood maxima when voting is independent. Finally, we show that the model's assumptions also imply the existence and uniqueness of electoral equilibria.

T. R. ____ (Reproduction in Process). Peter Coughlin, "Equilibrium Policy Proposals with Abstentions, March 1, 1981.

ABSTRACT

This paper analyzes spatial models of electoral competitions with abstention and probabilistic voting in which candidates have directional or local strategy sets. It also includes the important case in which incumbents must defend the status quo. The results derived here provide necessary and

sufficient conditions for directional, convergent stationary and convergent local electoral equilibria. These conditions provide a method for finding such equilibria. They also provide general existence results for directional, stationary and local electoral equilibria. These are pure strategy equilibria and their existence follows without introducing any special concavity or radial symmetry assumptions.

T. R. (Reproduction in Process). William M. Gorman, "Aggregates, Activities and Overheads," September 1980.

ABSTRACT

It is sometimes argued that the central office in a large organization should deal in aggregates, leaving decisions on detail to the periphery. This is the line taken, asking what the operational technology should be like to make such a procedure economic. Early work in this field focused on the aggregate production function for the economy as a whole, asking when it might be assumed to exist, and how to construct the measures of the quantities of 'capital', 'labor' and the like which enter into it. This paper suggests that the common element in all this work is that the aggregates can be considered as intermediate goods, 'produced' from fixed inputs, and themselves setting the conditions under which the variable goods are produced. This distinction between fixed input, inherited from the past and possibly badly adapted to current conditions, and variable goods, efficiently produced given the fixed inputs, and hence, having the same shadow prices everywhere, underlies all the work in this paper. One lathe may be lying idle in Birmingham, England, while another is working 24 hours a day in Birmingham, Alabama, where it is the effective constraint on production. Physically identical pieces of capital equipment in different locations can be poles apart economically so that there is no point in adding them together.

T. R. (Reproduction in Process). R. Aumann, Y. Katznelson, R. Radner, R. Rosenthal, and B. Weiss, "Approximate Purification of Mixed Strategies," March 1981.

ABSTRACT

Sufficient conditions are established for a finite game's Nash equilibria in mixed strategies to have corresponding epsilon equilibria in pure strategies. The principal condition is that each player's private information, conditional on any other player's private information, has an atomless probability distribution.

Randomized strategies play a significant role in the theory of games, but have limited appeal in many practical situations. Practical situations,

however, often possess enough exogenous uncertainty to render deliberate randomization intuitively unnecessary. In this paper we bring these observations together by describing conditions under which randomized strategies can be replaced by "approximately equivalent" pure strategies.

Suppose that n persons play a game in which each makes an observation (which may be related to the payoff) before play begins. While the observations need not be independent, suppose that even after making their observations and pooling the information so obtained, players 2, ..., n still cannot ascribe positive probability to any particular possible observation of Player 1. Then we will show that any mixed strategy of Player 1 can be epsilon purified, i. e., replaced by a pure strategy that yields all players approximately the same payoff as the original mixed strategy, no matter what strategies Players 2, ..., n use.

Next, suppose only that from his own observation no individual player can ascribe positive probability to any particular observation of any other player (a weaker hypothesis than if the information is pooled). Then we will show that every Nash equilibrium point in mixed strategies can be replaced by an n -tuple of pure strategies that is an approximate equilibrium (no player can gain much by deviating), and yields all players approximately the same payoff as the original equilibrium point.

Both of these results follow from our basic result, which is simply the two-person case of the first of the two results just described. Since the basic result deals with two-person games, the matter of pooling of information does not arise.

T.R. (Reproduction in Process). David Kreps and Robert Wilson, "Sequential Equilibria," January 1981.

ABSTRACT

This paper proposes a new criterion for equilibria of extensive games with perfect recall. The criterion, called sequential rationality, requires that each choice by a player be part of an optimal strategy for the remainder of the game. This criterion is stronger than Nash's and weaker than Selten's criterion for a perfect equilibrium. Consistency conditions are established for probability assessments not implied by Bayes' rule. Topological properties are established and it is shown that generically (in the payoffs) the equilibrium payoff distributions from sequential equilibria are finite in number and coincide with those of the perfect equilibria.

T. R. (Reproduction in Process). Herve Raynaud, "Paradoxical Results from Inada's Conditions for Majority Rule,"

ABSTRACT

In order to measure the restriction of liberty imposed on the voter by the value restriction conditions, we have counted the maximum number of different votes in a profile following single cavedness, single peakedness or the not-in-the-middle condition, which used to be confused with the bipartition condition.

It is shown in this paper that the two are not equivalent, that not-in-the-middle but no bipartition implies the appearance of a special design in the profile, and that in spite of their outlook, from the individual point of view, the pre-quoted conditions do not allow more freedom in voting than Black's condition.

T. R. (Reproduction in Process). Herve Raynaud, "Conditions for Transitivity of Majority Rule with Algorithmic Interpretations."

ABSTRACT

Using particular cases of the condition to be found in G. Kohler's thesis (Grenoble 1978), one can derive nine conditions for transitivity of the method of majority decision which all possess a generating algorithm with possible sociological or economical interpretations.

- Some celebrated conditions appear to be particular cases of this structure.
- Some other, "new" conditions, at least new in their algorithmic form, suggest explanations for some efficient committee voting procedures.
- The ninth and last one has the strange property of being impossible to satisfy for five objects or more.

T. R. (Reproduction in Process) Michael Taksar, "Infinite Excessive and Invariant Measures," November 1980.

ABSTRACT

In an earlier paper the following problem was considered. Given a contraction semigroup T_t on a Borel space D and an excessive measure v , when it is possible to find another contraction semigroup T_t' such that $T_t' > T_t$ and v is invariant with respect to T_t' ? The most restrictive condition under which this problem was solved is the finiteness of the excessive measure v . This condition excludes such an interesting case as the semigroup T_t generated by the transition function of Wiener's process and

the Lebesgue measure ν . In the present paper we extend the results to all quasi-finite null-excessive measures ν .

T. R. ____ (Reproduction in Process). Michael Taksar, "Enhancing of Semigroups."

ABSTRACT

$$(1.2.1) \quad \frac{\partial f}{\partial t} = Lf(t, \cdot) ,$$

$$(1.2.2) \quad Hf(t, \cdot) = 0 ,$$
$$f(0, x) = h(x) .$$

Here L is an elliptic differential operator of the second order in the space E , and H is a linear operator in the space of functions on E (this operator corresponds to the boundary conditions). Let \mathcal{D} be a space of twice continuously differentiable functions g on E such that $Hg = 0$ and let Q_t be a semigroup whose generator coincides with L on \mathcal{D} . Then $f(t, x) = Q_t h(x)$. The inequality $F(t, x) \leq h(x)$ shows that $h(x)$ is an excessive function with respect to Q_t . This is equivalent to $\nu_t(dx) = h(x)dx$ being an excessive measure with respect to the conjugate semigroup T_t .

We consider changing the boundary by changing the operator L in (1.2.1) into $L + L$, where L is an integral operator, and changing boundary conditions, that is, replacing H in (1.2.2) by a new operator \tilde{H} . Let \tilde{Q}_t be the new semigroup corresponding to the solution of the new system and \tilde{T}_t its conjugate. If under the new boundary conditions $f(t, x) = h(x)$ for all t , then ν is an invariant measure with respect to \tilde{T}_t . The problem is to construct the new boundary to ensure this property.

WORKING PAPERS

W. P. No. 2. Peter Coughlin, "Some Elementary Restatements of Increasing Marginal Revenue," March 1981

ABSTRACT

This note shows that the statement that a monopolist's marginal revenue function is increasing or equivalent to certain specific statements about the elasticities of the marginal revenue function, the slope of the aggregate demand function and the function $1 + e_{qp}$ (where e_{qp} is the price-elasticity of the aggregate demand function).

W. P. No. 3. Peter Coughlin, "Necessary and Sufficient Conditions for δ -Relative Majority Voting Equilibria," August 1980.

ABSTRACT

This note solves a problem which was recently posed and partially solved by Greenberg. To be specific, it derives the exact lower bound on δ which is necessary and sufficient for the existence of a " δ -relative majority equilibrium" for every possible profile in a given society. This bound is obtained under the conditions which Greenberg was concerned about in his results on δ -relative majority equilibria.

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